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(Millin, *Voyage dans les Départemens du Midi de la France;* Vaysse de Villiers, *Itinéraire Descriptif de la France;* Malte-Brun, *Géographie Universelle; Dictionnaire Géographique Universel.*)

VIERZON, a town in France, in the department of Cher, 111 miles in a direct line south or south by west of Paris, or 125 miles by the road through Orléans; in 47° 13′ N. lat. and 2° 3′ E. long. The town stands on the right bank of the Evre, just above its junction with the Cher, in a fertile plain. It is well-built; the houses are chiefly covered with slate. There are limestone-quarries and pipeclay and ochre pits in the neighbourhood; the ochre is considered the best in France. There are in the town iron and steel works, a porcelain manufactory and one for common earthenware, tan-yards, paper-mills, and manufactories for serge and woollen cloth. Trade is carried on in timber and wool: there are five yearly fairs, one of which is an important wool-fair. The population, including the adjacent village of Vierzon, which forms a separate commune, was, in 1831, 7967, of whom 4706 were in the town. There is an hospice or almshouse. (Malte-Brun, *Géographie; Dictionnaire Géographique Universel.*)

VIETA, FRANCIS. Much has been said of the writings of Vieta, but very little on his life, and that little has often been wrongly given. In the absence of all good sources of reference, we are under the necessity of giving somewhat more space to this biography than is usual. We also intend to insert in this article some account of Lucas Pacioli, which has been omitted in its proper place, and some additional details on Leonard of Pisa.

François Viet, Viette, or de Viette* (his name is given in these ways, and in one of his own writings it is Latinized Fr. Vietæus, but more usually Vieta), was born at Fontenaile-Comte, a small town not far from La Rochelle, in the year 1540. His family, if we may judge from the position which he occupied during the greater part of his life, must have had both rank and interest. We may connect the epoch of his birth with other parts of the history of science, by stating that he was born about the time when algebra was introduced into the northern parts of Europe from Italy, in the 39th year of the age of Cardan, and three years before the death of Copernicus: while Napier, Harriot, and Galileo were respectively 10, 20, and 24 years his juniors. Of his education and early years we know nothing, and the scanty materials for the rest of his life are found principally in the work† of his friend the president De Thou (*Hist.*, lib. cxxix.). Bayle charges this celebrated writer (*Dict.*, art. 'Rasario') with inaccuracy in his accounts of learned men: if we may disregard this imputation in the case of Vieta, with whom the biographer was personally and intimately acquainted, we cannot all the more help wishing that the facts preserved had been more in number, and of somewhat closer connexion with the scientific pursuits of Vieta. The whole of De Thou's account does not amount to more than a few insulated anecdotes, which are often repeated; and the want of information from other quarters respecting one of the greatest mathematicians of the sixteenth century may be accounted for if we remember the troubled times in which he lived, and the rule which he appears to have followed of printing all his works at his own expense, and distributing them as presents among his friends. This has been found almost uniformly to be a successful mode of preventing or diminishing posthumous fame.

The life of Vieta was passed in the public service: on the resignation of De Thou, he was made master of requests. We have seen it said that he held this office under Henry III., and elsewhere that it was in the household of Margaret, wife of Henry IV. Both statements are probably true, since De Thou assures us that his attention to the mathematics was only the relaxation of a whole life spent in public business, for which, says the historian, he had both talent and industry. And Vieta himself, in his answer to Adrian Romanus, says that he cannot profess to be a mathematician, but only a person to whom mathematical studies are delightful when he has leisure.* He lived and held office through the religious troubles of the reigns of Henry III. and Henry IV.: a letter of his friend Ghetaldi, hereinafter mentioned, proves that he was on the council of state in the latter reign, and we must suppose that his love of study induced him to confine himself to the simple duties of his calling. It seems however that he did not entirely escape the dangers of the time, or the attacks of the opposite party. In his dedication to Catherine de Parthenai, Duchesse de Rohan, and mother of the Duc de Rohan, well known as the leader of the French Protestants in the time of Louis XIII., he addresses that lady as one who had saved him from imprisonment and certain death: which means, we suppose, that he had fallen into the hands of the Huguenots. He proceeds to aver, but whether this be fact or dedication we have no means of knowing, that it was her love for and great skill in mathematics which first incited him to that study. Her literary attainments are mentioned by her biographers, and the account given by Vieta may be perfectly true. There is only one story in De Thou of his political services: —The extent and scattered character of the Spanish dominions having rendered their communications insecure in time of war, a cipher was invented with more than 500 characters, and these not permanently retaining the same signification. The complexity of this method foiled the ordinary decipherers, and application was thereupon made to Vieta, who without any difficulty discovered the secret, which was used for more than two years, to the great loss and annoyance of the Spaniards. These, perceiving that their cipher was detected, and imagining that no human skill was equal to such an effort, attributed the discovery to magic, and took care to publish this report throughout Europe, but particularly at the court of Rome. But the imputation failed to excite any odium, and was received, says De Thou, *non sine risu et indignatione rectius sentientium:* heresy had taken the place of sorcery. It is therefore not true, though some writers have said it by way of mending the story, that Vieta was actually cited to appear at Rome and answer the charge of dealing with the foul fiend.

Indirectly connected with the politics of the day is the share which Vieta took in the controversy on the reformation of the calendar. This, as is well known, was completed under the auspices of Pope Gregory XIII., in 1582, though the subject had been in agitation more than a century, and the change had even been projected by Sextus IV., in 1474. The plan finally adopted was that of Lilius,† an astronomer of Calabria, who died before its presentation to the pope, and the execution of it was intrusted to the Jesuit CLAVIUS. It is to be remembered that the true time of keeping Easter was then thought of the utmost importance, and that heterodoxy in this particular had more than once been thought worthy of excommunication. The reformed calendar was attacked by Vieta, Joseph Scaliger, and others, the first of whom published in the year 1600 what he called the true Gregorian calendar, and prefixed to it the bull of Gregory XIII. On this work it will be sufficient to say that Montucla and Delambre unite in condemning the ideas of Vieta: he made 3400 Julian years contain exactly 42,053 lunations, the error of which is a trifle more than that of the astronomy of his day. His work was carried by himself to Cardinal Aldobrandini, who was then at Leyden on a mission from Clement VIII. He had however no success with the cardinal, 'as I warned him when he set out,' says De Thou, 'feeling sure that an improvement adopted by the princes of Christendom after so much deliberation, would not easily be modified, even for the better, by those who think it a secret of government never to confess that they either have erred or can err.' Clavius simply replied to Vieta by referring him to a work on the Gregorian calendar which he was then preparing, and which he stated would contain a full reply to all the objections. This answer seems to have enraged Vieta beyond his powers of forbearance. Perhaps he felt indignant at not being considered worthy of a separate reply, or perhaps the malady which afterwards

* Albert Girard, at the beginning of the seventeenth century, and De L'Hopital at the end, both call him *Viette*.

† We have examined what Teissier has added in his collection of De Thou's biographies, and find nothing particular except the assertion that letters from Vieta are found in the collection of Caselius, which is totally inaccurate.

* His great contemporary Napier made a profession of the same sort. The interpretation of the Revelations and the overthrow of the Pope were his occupations; the mathematics, his relaxation.

† Libri corrects a mistake very often made, namely, the statement that Lilio was of Verona, and also the confusion between him and *Giglio* Gregorio Giraldi, frequently called *Lilio*, a learned writer, who published a work on the ancient calendar, but who died about 1552, long before the Gregorian reformation.

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destroyed him had begun to act upon his mind—which last may be charitably hoped. In 1602 he published his expostulation against Clavius, a tract of three pages, which Montucla is surprised his editors should have permitted to descend to posterity. He charges his opponent with evasion, and asserts that he ought to have retracted his error for the sake of the mysteries of religion, the peace of Christendom, and the divine authority of the supreme pontiff. He accuses Clavius of having slandered him to the pope, of contempt of religion, of falsehood in mathematics and theology; and urges upon him the danger that the Protestants might, through his obstinacy, get hold of the real calendar (his own) by themselves, and not from the papal authority. He calls upon Clement to alter the bull of his predecessor, and brings forward, curiously enough, as a precedent, that Augustus Cæsar, a Pontifex Maximus, had changed the arrangement of the year ordained by Julius Cæsar, another Pontifex Maximus. Finally, in order that no manifestation of bad feeling might be wanting, he calls upon the order of Jesuits to excommunicate all who should by design and fraud stand in the way of the good of Christendom; meaning, of course, Clavius and his followers. To this explosion of passion Clavius did not condescend to reply: but throughout his work, which appeared in 1603, the year of Vieta's death, he treated the latter with the respect due to his genius. De Thou gives a partial friend's account of this controversy, for he says that on the refusal of Clavius to adopt the emendations of Vieta, the latter sent him a serious expostulation, and that had Vieta lived, the matter would not have stopped there, since those who did not hesitate to pluck at the beard of a dead man, would have beaten the living one, had they dared. The anonymous author of the life of Vieta in the 'Biographie Universelle' has followed De Thou in the preceding description of the controversy, probably from having never seen anything but copies of this description.

It can hardly be supposed that so severe an attack upon the bull of Gregory XIII. would pass altogether unnoticed at Rome; and the treatment of Galileo, which was not many years after Vieta's death, may lead to a suspicion that, if Vieta had not died opportunely, he would have been compelled to desist from his opposition; and certainly, if the Inquisition had caught him on this matter, he would not, after the hint which he had thrown out about Clavius, have had the sympathy which posterity, with one voice, has expressed for Galileo. There is a circumstance which seems to us to make it probable that the storm was brewing. In 1603, just before Vieta's death, Theodosius Rubeus (author of a work called 'Diarium Universale,' published in 1581, and which seems to have been reprinted with additions in 1693), an ecclesiastic at Rome, published, 'permissu superiorum,' an expostulation* against Vieta on behalf of Clavius. This expostulation was dedicated to the pope, in terms which, unless used by permission, were presumptuous in the highest degree: since they certainly imply that the writer was empowered to say that recourse would be had to authority, if that expostulation were not sufficient. As this tract is never cited, and not easily obtained, we give at length the passage to which we allude:—'Itaque cum apud te solum, Pater Beatissime, hæc causa, cujus cognitio tua est, sit agitanda, censui *sub angustissimo nomine tuo*, hanc meam admonitionem in publicum dare, ut omnis provocandi ansa Vietæ tollatur, *et tandem huic controversiæ auctoritate tuâ finis imponatur.*' Rubeus afterwards pays a high testimony to the extent of Vieta's acquirements, which is well confirmed by such scattered notices of him as exist. He says that he feels it necessary to speak strongly in behalf of Clavius, since the latter is contending single-handed with one who is both lawyer, theologian, mathematician, orator, and poet.

What more we have to say of Vieta must appear in connection with his friendships or his writings. He died at Paris in 1603, according to De Thou: Weidler says December 13, but without stating from whence. Of his attachment to study the former writer says it was so excessive, that he often continued for three days together, fixed in thought, without stirring from his chair, or taking more sustenance or sleep than nature absolutely required. In religion he appears to have been a zealous Catholic, at least towards the end of his life, and in politics a confirmed believer in the divine right of kings. The assassination of Henry III. seems to have dwelt upon his mind for years, so much as to force him to recur to it in his writings, in places where political allusion is a curious kind of digression. Thus, at the end of his 'Responsa Mathematica,' published in 1593, he suddenly breaks off from the subject of the Calendar to refer to that event, which took place in 1589: 'Sed de iis tollendis ad ecclesiasticos referam commodiore loco, ac ipsis detegam portendum quæ summo ipsorum applausu mirum solis et lunæ consensum prodat εἰς ἱερὰ ἐπιμήνια. Sed,

'Eheu! quis unctum chrismate mystico
Necare regem, sacrilegá manu,
Ausus cucullatus sodalis
In numerum colitur Deorum!

Pii haud vacillent, ECCE MALUS BONIS.
Tremant procaces, ECCE BONUS MALIS
Non compater nomen sodali
Omen at imposuit nefando.'

The allusion in the verses is to Jacques Clement, who, after the assassination of the king, was considered as a saint by his party.

This article is the proper place of reference to several minor mathematicians, who are hardly worth separate articles in any except a very full biographical dictionary, but who owe some of their fame to their connection with Vieta. We may instance Nathaniel Torporley, Adrian van Roomen, Marino Ghetaldi, and Alexander Anderson.

Nathaniel Torporley, born about 1573, entered at Christ Church, Oxford, and after his degree was in France for several years: Wood says it is notorious that during that time he was amanuensis to the celebrated mathematician Francis Vieta. This fact has been mentioned by the French historians, in speaking of Harriot, when hard pressed to defend Des Cartes from the imputation of being Harriot's plagiarist; and the idea seems to be that as Torporley was afterwards under the patronage and in the house of Henry Percy, earl of Northumberland, as also were Harriot and others, he must have been in habits of intimate communication with Harriot, to whom he might have taught what he learnt from Vieta. With regard to the fact itself, it is almost certain, for not only does Wood mention it as notorious, but SHERBURNE, in the list at the end of his 'Manilius' (1675), published before Wood wrote, says that Torporley was 'sometime amanuensis to the famous Vieta.' Nothing is more likely than that Harriot learnt from Torporley many ideas of Vieta; but Harriot's discoveries in algebra most distinctly bear the mark of a new mind. Torporley afterwards wrote his 'Diclides Cœlometricæ, seu Valvæ Universales,' &c., London, 1602, and other works which we have never seen. Wood also says he wrote something against Vieta, under the name of Poulterey, a transposition (not perfect, however) of his own name, but which he (Wood) had never seen. In looking through the 'Diclides,' &c., which is mostly on spherical trigonometry, we only found two very slight notices of Vieta's name, which looks as if there had been a coolness between them; but we found, to our surprise, that Torporley had preceded Napier by twelve years in the publication of the greater part of the rule of CIRCULAR PARTS, not indeed in Napier's convenient form, but with a complete reduction of the six cases to two, and rules, such as they were, by which to assimilate the connected cases. For more account of Torporley's process, which is the greatest burlesque on mnemonics we ever saw, we refer to the 'Philosophical Magazine' for May, 1843. We have only to add that Torporley obtained church preferment, was a member of Sion College (to which he left his books and manuscripts), and died in April, 1632. In the Catalogue of Sion Library it is said he was a chemist who left a large number of chemical and other books; but we cannot find one of his works in the second catalogue, and we have not had the opportunity of examining the first. The fire of London occurred between the publication of the two, and the books which were then consumed are not mentioned in the second.

Adrian van Roomen, commonly called Adrianus Romanus, born at Louvain, September 29, 1561, died May 4, 1615 (1625?). He published various works, of which the names may be found in Vossius 'De Scientiis Mathematicis.' The story of his acquaintance with Vieta is told by De Thou, but more in detail by Tallemant des Réaux, whose 'Historiettes' (written before 1657) were lately pub-

* We never saw any mention of this work, except in a manuscript cross-reference from 'Vieta' in the catalogue of the British Museum.

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lished at Paris (1834-35, 6 vols. 8vo.). In his 'Idea Mathematicæ,' &c., Antwerp, 1593, Romanus proposed a problem to all the celebrated mathematicians whom he knew by reputation, naming them, but without a French man among them. Shortly after, the ambassador of the States being at Fontainebleau, in conversation with Henry IV., who was enumerating to him the celebrated men of the country, said, 'But, Sire, you have not a mathematician, for Adrian van Roomen does not name one Frenchman in his list.' 'Indeed I have, though,' answered the king, 'and an excellent one—let some one call M. Viète.' Vieta came, was presented to the ambassador, who gave him Van Roomen's problem, placed himself at a window, and, before the king left the room, wrote two solutions with a pencil. In the evening he sent several others, offering more, as he said the problem was capable of any number. Van Roomen, immediately on hearing of this, set off to Paris to see Vieta, followed him to Fontenay, and spent some weeks with him. We shall see more of his problem presently. Tallemant, who was evidently not a mathematician, tells us the sort of impression which Vieta's writings had created about the middle of the seventeenth century. He says that this M. Viète, who had learnt mathematics by himself, there being nobody to teach him in France, wrote treatises so difficult that no one of his age could understand him: that one Lansberg, if he mistakes not (but he does mistake), first deciphered some of them, and that since his time people had made out the rest. It is worth noting that this same Tallemant is a witness independent of De Thou, for he informs us that Vieta died young, of study, whereas, had he seen De Thou's account, he would have found in the very first words that Vieta died 'anno climacterico.' And yet Alexander Anderson, who must have known his friend's age, calls his death 'fatum immaturum.'

Marino Ghetaldi, of Ragusa, was of a good family, but of his life* we can find nothing; nor of his death, except that it took place before 1630. Tallemant, already cited, says that a Ragusan gentleman, called Galtade (Ghetaldi), procured himself to be made minister of his native republic in France, that he might have the acquaintance of Vieta. Ghetaldi, in the letter already alluded to, says he was at Paris on his own affairs when he first met with Vieta. The works of Marino Ghetaldi are—1, Rome, 1603, 'Nonnullæ Propositiones de Parabola;' 2, Rome, 1603, 'Promotus Archimedes,' a work on specific gravities, which is sometimes cited on matters of weights and measures; 3, Venice, 1607, 'Apollonius Redivivus;' 4, Venice, 1607, 'Supplementum Apollonii Galli,' in continuation of the tract of Vieta presently mentioned; 5, Venice, 1613, 'Apollonius Redidivus' (the second book); 6, Venice, 1607, 'Variorum Problematum Collectio;' 7, Rome, 1630 (posthumous), 'De Resolutione et Compositione Mathematica,' folio, all the others being quarto). There is not much of algebra in Ghetaldi's writings, but what there is comes from the school of Vieta: the author so far bears out Tallemant's story, that he speaks of his intimate friendship with Vieta at Paris.

Alexander Anderson, born at Aberdeen in 1582, taught mathematics publicly at Paris, and was the editor of two of Vieta's works, which came into his hands, one from the author, the other from his executors, as will presently appear. A list of his works, and an abstract, by Mr. T. S. Davies, will be found in the appendix to the 'Ladies' Diary' for 1840. Both Ghetaldi and Anderson defended a solution of Vieta from the attack of a certain Clemens Cyriacus in 1616. (See the Society's *Biographical Dictionary*, 'Anderson.')

It may perhaps save some bibliographical student a hunt for an imaginary work of Vieta if we mention here the 'Supplementum Fr. Vietæ, ac Geometriæ totius Instauratio,' Paris, 1644, by A. S. L. This A. S. L. is Antonio Sanctini of Lucca, who had a few years before published 'Inclinationum Appendix,' &c., with his name. At the head of his dedication he calls himself *Constantius Silanius Nicenus*, which is an anagram for *Antonius Sanctinius Lucensis*. The work itself is an impudent attempt to connect Vieta's name with pretended solutions of the problem of two mean proportionals, the multisection of the angle, &c. Both Sanctini's works were answered by P. P. Caravaggi of Milan, in his 'In Geometria, &c. Rimæ detectæ,'

* Morhof (*Polyhistor*, ii. 473, edition of Fabricius) gives a reference to the Life of Father Paul Sarpi, in which Ghetaldi is mentioned, perhaps with some account.

&c., Milan, 1650. Sanctini's algebra is of the school of Vieta. It is a striking corroboration of what may be suspected for other reasons, namely, how little Vieta was appreciated in France for many years after his death, that of all the persons we have mentioned as connected with him, not one is a Frenchman; but nevertheless some part of his works was translated into French by one Vaulezard; we know that this translation exists, but we cannot find any mention of it.

The writings of Vieta are rendered difficult to read by the then almost universal affectation of forming new terms from the Greek, and of introducing phrases in that language. His pages may remind the reader of the English fashionable novels of ten years ago, which required a continual insertion of French words and sentences. Thus, in the *isagoge*, we find *zetetic*, *poristic*, and *exegetic* processes, the first consisting of *antithesis*, *hypobibasm*, and *parabolism*; and also that by an additional axiom, 'αἴτημα non δυσμήχανον,' many problems hitherto 'ἄλογα,' may be solved 'ἐντέχνως.' He uses the signs + and −, and also that for division: but when he would designate the difference of two quantities of which the greater is unknown, he places between them our modern sign of equality, thus: A = B. The exponents are expressed by words, either full or contracted; and the numerical coefficients are written after their accompanying letters. The analogy between algebra and geometry, which gave the name of square and cube to the second and third powers, is extended to all symbols. Thus the equation $3BA^2 - DA - A^3 = Z$, would be written

B 3 in A quad. — D plano in A — A cubo equatur Z solido.

Here D is called D *planum*, and is considered as the representative of a geometrical superficies, that the second term may be homogeneous with the first: for a similar reason Z is *Z solidum*. And in various places it is expressly laid down that it is not allowable to compare quantities which are not thus rendered homogeneous. The great difference between the methods of Vieta and of his predecessors is one in which lies much, if not the greater part, of the power of algebra: he was the first who used letters to signify known or determinate quantities, and he was the first who systematically combined the use of symbols of quantity with that of symbols of operation. By this method, the comprehension of a process which expressed in words would be long and complicated, does not cost the practised eye a second glance. It is true that the operations of those who preceded Vieta would lead to a correct numerical result in any particular case: but the result only appeared, and the *modus operandi* was either lost or wrapped in the dusky folds of a verbal rule. The notation of Vieta expresses at once the rule and the result, and is a step in the advance of science which, for the magnitude of its consequences, deserves to be ranked with the invention of fluxions. There is much truth in the remark of Vieta upon his predecessors: 'Vovebant Hecatombas, et sacra Musis parabant et Apollini, si quis unum vel alterum problema extulisset, ex talium ordine qualium decadas et eicadas ultrò exhibemus, ut est ars nostra mathematum omnium inventrix certissima.'

We now proceed to a short account of the writings of Vieta, referring for more detail to the second volume of Hutton's tracts. Vieta, as we have said, printed his works privately, and we are not wholly able to recover the dates of the several first publications.

[But* it is not noticed that many of these works, which are now only known by the edition of Schooten, were published together, or at least preceding publications were joined together in one, by Vieta himself, before the year 1591, under the name of 'Restituta Mathematica Analysis, seu Algebra Nova.' Neither Montucla, nor any other modern writer that we have seen, appears to be aware of this fact: the French historian does not seem to know that the first seven books of the 'Responsa Mathematica,' of which (i. 578) he regrets the loss, were contained in the collection alluded to. The fact is nevertheless certain, as the following editions of different separate works—viz. 'In Artem Analyticam Isagoge,' Tours, 1591; 'De Numerosa Potestatum ad Exegesin Resolutione,' Paris, 1600; and 'Supplementum Geometriæ,' Tours, 1593;—contain in their title-pages the name of the source from whence they were taken, and the first of them also gives a list of the contents, from which

* We put this paragraph in brackets, as we first wrote it, for a reason afterwards mentioned.

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list we have placed R. M. before the titles of the following descriptions, in every case in which the 'Restituta Mathematica' is said to have contained the work. Besides these, we must reckon among the contents the seven first books of the *Responsa*, which have not come down to us, though tradition has preserved the name; and 'Ad logisticen speciosam notæ posteriores,' of which even the very name has disappeared from the history of algebra. We cannot help hoping that some old library may yet be found to contain this collection. Other writers take the words of the title in a sense between that of quotation and description. Thus Alexander Anderson says, '*Restitutam Mathematicam Analysin* F. Vietæ debetis, φιλομαθεῖς.' And Walter Warner (preface to Harriot), '*Artis Analyticæ Restitutionem* F. Vieta aggressus est.']

We believe it will be shorter and clearer to leave the preceding passage in brackets (for which we thought we had very fair evidence), and to make a suspected correction, as another writer would do; in preference to mixing up the mistake (if it be a mistake) and the correction. The first publication of the 'Isagoge,' &c. (1591) bears on its title-page that it is 'Seorsim excussa ab *Opere* Restitutæ Mathematicæ Analyseos, seu Algebræ Novæ :' and on the reverse of the title-page appears 'Opere Restitutæ Mathematicæ Analyseos, seu Algebrâ Novâ, continentur . . . . : *Operi* autem Preposita *est* sequens epistola.' Ten works are given by title, which may, all but the *seven books* and the *notæ posteriores* already noticed, be collected from the indication (R. M.) in the following list; and the epistle is the dedication to Catherine of Parthenai before alluded to. Blancanus (1615) places 'Opus Restitutæ,' &c. in the list of Vieta's works; and Morhof says that Vieta wrote 'Isagoge, &c. *seu Algebra Nova*.' Can any evidence be more positive to the fact that a work was published, or at least written out for publication? The absence of date or printer's name tells nothing as to that period, for books were then few, and did not require the minute accuracy of description which is now necessary to distinguish one work from another: moreover, whether this be the reason or not, such accuracy of description was not usual. Why then do we not continue to believe that such a work was published? In the first place it is entirely lost, and with it the *Responsa* and the *notæ posteriores*, which is not likely to have happened to a large collection of Vieta's works: in the second place, Anderson, in his publication (which he gives us to understand was the first that was made) of the treatise 'De Recognitione,' &c., tells us something about Vieta's habits, which seems to explain the whole. 'He was,' says Anderson, 'in the habit of [referring to as finished' (*insignire* solebat) and by their names, works which, though undertaken in his own mind, and digested in order, were not even so much as fairly written down, owing to the interruption which his studies received from his public duties. This, then, may be the whole secret: Vieta gave a list of the works which he intended to publish, under the name which he intended to give them collectively. The seven books of the *Responsa* and the *notæ posteriores* never, on this supposition, were published at all. And it will afterwards appear that there was a reason why the eighth book of the *Responsa* should have been published without the rest; though it is singular, if the list above named be only of works intended, that this eighth book, which must have been as finished as the rest, should not have been mentioned. It is almost incredible, moreover, that Alexander Anderson should have published a few of Vieta's theorems, with his own demonstrations, as new, if Vieta had published them, and more, twenty years before.

(R. M.) *In Artem Analyticam Isagoge*, first published by Vieta himself, at Tours, in 1591. Here are laid down the principles of homogeneity before alluded to, and the common axioms used in the solution of simple equations. Many new terms are introduced, of which only two have lasted, namely, the distinction of equations into *pure* and *adfected*. The law of homogeneity is a fanciful deduction from certain well-known analogies between arithmetic and geometry, and the manner in which it is applied renders this book of Vieta somewhat obscure. The following is a specimen: 'Lineam rectam curvæ non comparat (probably corrupt, *comparare non licet*), quia angulus est medium quiddam inter lineam rectam et planam figuram. Repugnare itaque videtur homogeneorum lex.'

(R. M.) *Ad logisticen speciosam notæ priores*. The *notæ posteriores*, as just mentioned, are lost. *Logistice Speciosa* is the literal algebra, as distinguished from *logistice numerosa*, or common arithmetic. Here are various questions in algebraical addition and multiplication: the powers of a binomial are raised up to the sixth inclusive, and the law of the exponents is given, but not that of the coefficients. Particular notice is taken of the addition of powers of A+B and A−B, and, in a few cases, of the composition of A^n-B^n. Various methods are given of forming right-angled triangles whose sides shall be whole numbers.

(R. M.) *Zeteticorum libri quinque*. The first book contains problems producing simple equations, of which the following are specimens:—Given $x\pm y$, $x\pm z$, and the ratio of y to z, to find x; given the sum or difference of two numbers, and of given proportions of those numbers, to find the numbers. Here, as elsewhere, Vieta uses the capital letters only, and represents the unknown quantities by vowels, and the known quantities by consonants. The second book is full of those problems of the second and third degree, which produce unadfected equations, solved as in our modern works. The third book contains the reduction into equations, and solution, of questions in proportion, and also of right-angled triangles. The fourth and fifth books give the solutions of various of those problems now called Diophantine; mostly collected from Diophantus himself. We find here the first use of the vinculum connecting terms whose result is considered as a whole. Blancanus says that Cataldi explained this work of Vieta in what he calls 'continuatio algebræ proportionalis,' which cannot be the 'nova algebra proportionale,' Bologna, 1619, published after Blancanus wrote.

(R. M. as to the first, not the second.) *De Equationum Recognitione et Emendatione libri duo*. First put together by Alexander Anderson, who obtained the materials from Alelmus or Aléaume (who had charge of Vieta's papers), and published these books at Paris in 1615. The first six chapters of the treatise *De Recognitione* are employed in demonstrating that equations of the second and third degree spring from questions upon three and four continued proportionals, except in the irreducible case of the latter species, which is shown to depend on the trisection of an angle. Where a cubic equation has one root only, and that negative, the equation is deduced which has the corresponding positive root. The two roots of an equation of which one is negative are not considered, but the equation is deduced which has a positive root corresponding to the negative root of the former, and this equation is called contradictory to the former. Various methods are found by which an equation of a higher degree may be deduced from a given one, a synthetical process apparently introductory to the subsequent depression of equations. In the treatise *de Emendatione* Vieta lays down rules for destroying the second term of an equation of the second or third degree. He then shows, in a cubic equation which has the highest term negative, how to avoid this by a transformation which is in effect finding the equation whose roots are reciprocals to the roots of the former equation. We have not space to enter minutely into the various transformations: we will only remark generally that an equation is considered unfit for use in which the highest power of the unknown quantity is negative, or has a coefficient, and that the greater part of the reductions employed would not be necessary to a modern analyst. These books leave the reader in possession of the methods then known for the depression or solution of equations of the second, third, and fourth degrees. They are a luxuriant exercise of the power newly derived from Vieta's improvements in notation. He concludes by showing how to construct an equation which shall have given positive roots, which forms the suggestive basis of the subsequent discoveries of Harriot. On this he observes, 'Atque hæc elegans et perpulcræ speculationis sylloge, tractatui alioquin effuso, finem aliquem et Coronida tandem imponito.' Dr. Hutton mistranslates when (*Hist. Alg. Tracts*, vol. ii) he concludes from these words that Vieta only announces the theorem, 'and for this strange reason, that he might at length bring his work to a conclusion.' Nevertheless, Hutton's account is generally a very good one.

(R. M.) *De Numerosa Potestatum purarum atque adfectarum ad exegesin resolutione tractatus*. This work, first published, with Vieta's consent, at Paris in 1600, has at the end a letter (hereinbefore referred to) from Ghetaldi to Michael Coignet, a Belgian mathematician, who states that at his earnest entreaty Vieta had consented to allow the work

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to be published, on condition that he (Ghetaldi) would take the trouble of editing it. This letter mentions the seven books of the *Responsa*, the *Harmonicon Cœleste*, &c. The *numerose exegesis*, as the method herein explained was frequently denominated, is given, with the most modern improvements, in the article INVOLUTION AND EVOLUTION, and its history will be found in the 'Companion to the Almanac' for 1839. It passed through the hands of Harriot, Oughtred, and Wallis, with some improvements, but was so prolix, and required so much calculation, that when Newton's method appeared [APPROXIMATION; THEORY OF EQUATIONS] it gradually sank out of use. The late Mr. Horner of Bath, reproduced it, with a capital improvement in the mode of making the successive computations, which will establish it permanently. Very recently, Mr. Thomas Weddle of Newcastle, author of 'A New &c. Method of solving Numerical Equations,' has produced the kindred method of finding the highest denomination of the root, and correcting it by successive *multiplications*, instead of additions: a method which has considerable advantages when the degree of the equation is high. To return to Vieta: when the root is irrational, and any given degree of approximation is required, instead of using fractions, the equation is found whose roots shall be ten, or a hundred, &c., times the root of the given equation, which roots are then extracted by the method within a unit. The introduction of our notation for decimal fractions had not taken place at the time we are speaking of, though we should not be justified in drawing this conclusion from the mere fact of not finding it used by Vieta. From his avocations, perhaps, but more from the imperfect modes of communication (for there were then no scientific associations), he appears not to have been perfectly aware of what was going on in other parts of the mathematical world. So that it is impossible to say, at present, whether some of the things which we know to have been discovered before his time, may not have been, as far as he knew, the fruits of his own investigation. 'He neglects to avail himself of the negative roots of Cardan' (but this however was done, on principle, and from a determined refusal of all symbolical extension), 'the numerical exponents of Stifelius, instead of which he uses the names of the powers themselves; or the fractional exponents of Stevinus; or the commodious way of prefixing the coefficient before the quantity; and such like circumstances; the want of which gives his algebra the appearance of an age much earlier than its own.' (Hutton, *Tracts*, ii., 273.) He had however seen the exponents of Stevinus, and the prefixed coefficients, for Van Roomen's problem, as given by himself, contains both.

(R. M.) *Effectionum Geometricarum Canonica Recensio* and *Supplementum Geometriæ*. The second of these works was first published at Tours in 1593. The former of these treatises is a collection of problems in common geometry, intended to facilitate the solution of problems of the second degree. The second treatise assumes the construction of the conchoid of Nicomedes; the finding of two mean proportionals, the trisection of an angle, the inscription of a regular heptagon in a circle, and the solution of the irreducible case of cubic equations, are made to follow. The last of these is contained in the following proposition:—'If there be two isosceles triangles, having the equal sides of one equal to those of the other, and the equal angles of the second triple of those of the first, the cube of the base of the first diminished by three times the parallelopiped under the base of the first, and the square of the common side, is equal to the parallelopiped under the base of the second and the square of the common side.'

Pseudo-mesolabum. The term *mesolabum* was applied to any process by which two mean proportionals could be found between two given straight lines. By *Pseudo*-mesolabum Vieta means a process which, though not limiting itself to Euclidean geometry, nevertheless is effective on its own suppositions. A chord of a circle cuts a diameter, and a perpendicular from one extremity of the chord cuts the diameter produced, so that the part produced is equal to the chord. This being the case, the segments of the chord are mean proportionals between those of the diameter. In the article DUPLICATION, &c., we have done Vieta wrong by imputing to him a great mistake in this matter. The fact is, that when he has finished his pseudo-solution (merely ungeometrical), he then is ambitious of showing how well he can reason falsely, and ends with a *pseudo*-theorema (meaning one which is avowedly untrue, and given to be afterwards exposed). Now if a man will write a pseudo-method, which he himself defines to mean no more than unallowed by Euclid, and makes his treatise to end in nothing but a *pseudo*-theorema (intended to be false), not even the closest examination will prevent every one from supposing that his pseudo-theorema is the *finis atque corona* of his pseudo-method.

(R. M., in which it is called *Analytica Angularium Sectionum in tres partes distributa*). *Ad Angulares Sectiones Theoremata καθολικώτερα*. This is really Alexander Anderson's publication. Vieta sent him the theorems, he found out the demonstrations, and published them, in 1615, at Paris, with a dedication to Charles, prince of Wales. Among many trigonometrical theorems are here given some of the class of which we shall presently speak with respect to Van Roomen's problem. The chord of an arc being given, the chords of its multiples and of their supplements are found.

Ad Problema quod omnibus mathematicis totius orbis construendum proposuit Adrianus Romanus Responsum. The circumstances under which Vieta first saw this problem have been already stated from Taillemant. It amounts to this: given the chord of an arc, to express algebraically the chord of the 45th part of that arc; but it is given in the form of a proposed equation of the 45th degree. If Vieta sat down at a window and solved several cases while Henry IV. and the Belgian ambassador were talking in the room, it must have been because he was then in full possession of his theory of angular sections, and saw at once that Van Roomen's problem was a particular case of it. But it must not be forgotten that the latter must also have been in possession of the same or of cases of it. This answer of Vieta is a full one, and appears to have been drawn up deliberately: he gives the complete reduction of the problem, with a good deal of what he must have supposed to be fun, but of a very ponderous and sober character. He ends by proposing, in his turn, a problem, evidently directed at Van Roomen, and by way of hit at his fearful equation and enormous coefficients, he says, 'Porro ad exercendum non cruciandum studiosorum ingenia, problema hujus modi construendum subjicio.' The problem is one of Apollonius, of which the solution had been lost,—Given three circles, to find a fourth touching them all.

Apollonius Gallus, seu exsuscitata 'Apollonii Pergæi περὶ ἐπαφῶν Geometria, first published by Vieta at Paris, in 1600, and addressed to Van Roomen. It has, in the beginning, a Greek epistle, anonymously addressed (perhaps by Van Roomen himself) Φραγκίσκῳ Ουιέτῃ, which is a presumption that the true pronunciation is Viēta. Van Roomen, as appears by the introduction, solved the preceding problem by the help of the hyperbola, on which Vieta rallies him in his manner, and proceeds to a geometrical solution. He then gives geometrical solutions of some problems which Regiomontanus had solved algebraically, but professed himself unable to solve geometrically. He calls himself Apollonius Gallus, and Van Roomen, Apollonius Belga; and from that time it became a fashion for those who had done anything after the manner of a particular Greek, to adopt the name of that Greek, with an adjective of country annexed. Thus Snell, after his measure of the earth, called himself Eratosthenes Batavus.

Variorum de Rebus Mathematicis Responsorum liber octavus. This book, first published at Tours in 1593, is preceded by an epistle from Pet. Da., whoever he may be, which explains why it appeared. It seems (at least it is so asserted) that there was at that time a great excitement at Tours, not only among the educated, but even down to the lowest of the people, about the quadrature of the circle, the problem of two mean proportionals, &c.; and Pet. Da., who had seen Vieta, and knew that he had a book on the subject lying by him, solicited and procured its publication. We have already spoken of the first seven books, which, if they were ever written, are lost. This book contains the history of, and remarks on, the method of finding two mean proportionals, various modes of applying mechanical curves to the quadrature of the circle, approximate solutions of the same problem, and a collection of formulæ for the solution of triangles, with a short chapter on the calendar.

Munimen adversus Nova Cyclometrica. This was a

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refutation of Joseph Scaliger's asserted quadrature of the circle, though the name of Scaliger is not mentioned in it. This eminent scholar was exceedingly angry, and attacked Vieta with much bitterness. But he afterwards, according to De Thou, changed his tone, admitted his error, and did justice to his opponent. Vieta himself had a high respect for Scaliger, as might be inferred from his suppression of the name. If Isaac Casaubon is to be trusted, he thought most highly even of the mathematical knowledge of Scaliger. In one of Casaubon's letters to De Thou (p. 307 of the collection), he says, that on one occasion he and a friend paid a visit to Vieta, and that, Scaliger's name coming up in conversation, Vieta said, 'I have so great an admiration of that astounding genius, that I should think he alone perfectly understands all mathematical writers, particularly those of the Greeks.' And he added, that he thought more highly of Scaliger when wrong than of many others when right.

Relatio Calendarii verè Gregoriani (Paris, 1600); *Kalendarium Gregorianum perpetuum*, and *Adversus Christophorum Clavium Expostulatio* (Paris, 1602). We have said enough of these unfortunate works in the preceding part of this article. The expostulation is preceded by Greek verses addressed to Clavius.

All the preceding works are contained, in the order in which we have mentioned them, in the collected edition of Vieta's works, edited by Schooten, and printed by the Elzevirs at Leyden, in 1646. It seems that Vieta's papers had either been almost entirely destroyed or else exhausted: for though the Elzevirs, in 1640, advertised their intention of printing such an edition (in the first number of the 'Catalogus Universalis,' an annual book-list, printed at Amsterdam), requesting those who had anything unpublished of Vieta's to communicate it, and giving the names (without dates, unfortunately) of all that had been published, yet they could not print, six years after this advertisement, one single treatise which did not appear in their own advertisement as already known. We have yet to speak of two other works, both remarkable in their way, which are not in Schooten's collection.

Harmonicon Cœleste.—This work has only been recovered in our own day. Schooten's reason for not giving it was, that he could only find an incomplete and inaccurate copy to print from: but he says that he had reason to suppose he should obtain a more complete copy, which he promised to publish with other writings of Vieta; no such work ever was produced. The very year before this preface of Schooten appeared, Bouillaud, in the prolegomena to his 'Astronomia Philolaica' (1645), says that Peter Dupuis (Petrus Puteanus) had lent the manuscript to Mersenne, and that some borrower, or more professed thief (but which is not said), had obtained it from Mersenne, and had never returned it. Some particular person is evidently pointed at: Bouillaud says this borrower would neither restore it nor a copy of it, and suspects that he meant to publish it as his own. Bouillaud was a good authority in this matter: he was known to De Thou, Schooten, &c., and Peter Dupuis was one of his colleagues in the formation of the catalogue of De Thou's library, and perhaps, if the story be true, got the manuscript out of that library to lend it to Mersenne. This story has been repeated in many English writers on this subject, from Sherburne down to Hutton, and always in the same words. Some inquiries which the writer of this article made some years ago at Paris through a most competent investigator, ended in the assurance that it was *in Bouillaud's handwriting* in the Royal Library at Paris, that he (Bouillaud) had himself lent the manuscript to Cosmo de' Medici of Tuscany, which must have been after it was recovered from Mersenne's honest friend, and of course after the publication of the 'Astronomia Philolaica.' Lately M. Libri (*Hist. des Sci. Math. en Italie*, vol. iv., p. 22) announces that there is an imperfect manuscript in the Royal Library at Paris, and that the original manuscript of Vieta (and an old copy, which however is mislaid) is in the Magliabechian Library at Florence (which confirms the last statement of Bouillaud). He gives a short account of the contents of the Paris manuscript, which contains various modifications of Ptolemy's theory, and sufficient proof that Vieta well knew both the writings of Copernicus and Tycho Brahé. Of the former he says that the excellence of his system, if any, is destroyed by the badness of the geometry by which it is explained; and M. Libri states that he avows his opposition to the heliocentric system still more plainly in other places. There is one conjecture which is worthy of some attention: we have seen how imperfect is the evidence for attributing to Apollonius the opinion afterwards maintained by Copernicus: Vieta asserts that this opinion was called Apollonian not because Apollonius promulgated it, but because the sun (Apollo) is in the centre of the system.

It was said that the 'Harmonicon Cœleste' was to be published, but it has not yet appeared.

Canon Mathematicus, seu ad Triangula, cum appendicibus, Lutetiæ, apud Johannem Mettayer, &c., 1579, to which is annexed, with a new title-page, 'Francisci Vietæi universalium Inspectionum ad Canonem Mathematicum liber singularis, Lutetiæ,' &c., as before.

This same book, from the same types, is also found with another title-page, as follows:—'Francisci Vietæi opera mathematica, in quibus tractatur canon mathematicus, seu ad triangula: item Canonion, &c. &c. &c., Londini, apud Franciscum Bouvier,* 1589.

The same book, again from the same types, is in the British Museum with a third title-page, as follows:—'Francisci Vietæi Libellorum Supplicum in Regia magistri, insignisque Mathematici, varia opera mathematica: in quibus tractatur Canon Mathematicus, seu ad triangula: item Canonion, &c., Parisiis, apud Bartholomæum Macæum, &c., 1609.

That the second and third are really the same book as the first, with a new title-page, we have ascertained by carefully comparing various words which are misspelt, and letters and lines which are broken, in all three: also by the fact that the second title-page, 'Francisci Vietæi, &c.' is the same, date and all, in the second. In the third the second title-page is taken out, and Mettayer's address is printed after the first. This book was, from its extreme scarceness, a bibliographical curiosity: we have seen five copies, three with the first title-page, one with the second, and one with the third: in two of the first three, some figures which are not found in the third have been stamped in after the printing; and the same stamping is apparent both in the fourth and fifth. The *canon mathematicus* is the first table in which sines and cosines, tangents and cotangents, secants and cosecants, are completely given: they are arranged in the modern form, in which each number entered has a double appellation. But the notation of decimal fractions not being invented, the mode of description is as follows:—to give the sine and cosine of 24° 2′, Vieta states that, the hypothenuse being 100,000, the perpendicular and base are 40,727 and 91,330 9: and in a similar way for the others: and here it is remarkable that in the cosines Vieta does use a species of decimal notation, leaving a blank space instead of using a decimal point; for, to an hypothenuse 100,000, the base to an angle of 24° 2′ is what we should now write 91330·9. There is also a large collection of rational-sided right-angled triangles, which form a trigonometrical canon, but not ascending by equal angles. The work concludes with a copious collection of trigonometrical formulæ and various numerical calculations, for mention of which see Hutton's 'History of Trigonometrical Tables,' prefixed to his logarithms, and inserted in his tracts. A short preface by Mettayer, prefixed to the 'Universalium Inspectionum,' &c., states that Vieta found great difficulties in getting tables printed at all, and also that plagiarists had printed and sold something of the kind, but what is not stated. Vieta himself (Schooten, p. 323) calls this book *infeliciter editus*, and hopes that a second edition will be of better authority.

Having now given, we believe, as complete an account of Vieta as existing materials can furnish, in consideration of the very meagre manner in which his biography is usually treated (the article in the 'Biographie Universelle' is very poor, considering that the work is French, and Vieta the greatest French mathematician of the sixteenth century), we may speak briefly upon the merit of his writings. Vieta is a name to which it matters little that we have not dwelt on several points which would have made a character for a less person, such as his completion of the cases of solution of right-angled spherical triangles, his expression for the approximate quadrature of the circle, his arithme-

* We cannot find the name of Bouvier in the list of English publishers of the sixteenth century, given in Johnson's 'Typographia.' In [illegible] instance is given (p. 497) of a foreign book being furnished with a London title-page.

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tical extensions of the same approximation, and so on. The two great pedestals on which his fame rests are his improvements in the form of algebra, which he first made to be a purely symbolical science, and showed to be capable of wide and easy application *in ordinary hands;* his application of his new algebra to the extension of trigonometry, in which he first discovered the important relations of multiple angles; and his extension of the antient rules for division and extraction of the square and cube roots to the *exegetic* process for the solution of all equations, which, with Mr. Horner's new mode of conducting the calculation, is becoming daily of more importance. He did not, as some of the French say, lay down the view of equations which was afterwards done by Harriot, but he gave strong suggestions towards it, stronger suggestions than the Italian algebraists had furnished him with for his own new algebra: it is Harriot's praise that he saw how to go on from where Vieta had stopped, as it is that of Vieta to have proceeded from the point at which Cardan had stopped. Neither did he, as some of the French again say (but not from national feeling in this instance), first apply algebra to geometry; for if by the application of algebra be meant the method of coordinates, that application is wholly due to Des Cartes, assisted, no doubt, by the power which Vieta conferred on algebra. But if nothing more be meant than the solution of geometrical problems by help of algebraical symbols and methods, many have claims before Vieta; for instance, Regiomontanus, Cardan, and Bombelli. Nay, Vieta himself points out that Regiomontanus had solved problems algebraically which he complained of not being afterwards able to do geometrically; and Vieta himself supplies the geometrical verification of Regiomontanus's algebraical solutions. Neither did he, as some of the French again say, show how to form the coefficients of the powers of a binomial: he saw, no doubt, the connection of them with the series 1, 2, 3, &c., 1, 3, 6, &c., 1, 4, 10, &c., as Tartaglia had done before him; but he did not show how to form them by any algebraical law, as Newton afterwards did. If a Persian or an Hindu, instructed in the modern European algebra, were to ask, 'Who, of all individual men, made the step which most distinctly marks the separation of the science which you now return to us from that which we delivered to you by the hands of Mohammed Ben Musa?' the answer must be—Vieta.

The earliest history of algebra is that contained in the mixed treatise of Wallis (in English, 1685; in Latin, 1693). Wallis had a partiality for Harriot which not only blinded him to much of the merit of Vieta, but furnished him with spectacles by which he could see most of the discoveries of the latter only in the writings of the former. Montucla has fairly and properly exposed this tendency: but that he may be disqualified to throw a stone at Wallis, he, in his turn, gravely and seriously declares that he cannot see the merit of the invention of *aa*, *aaa*, &c., to represent the powers of *a*, instead of Vieta's mode. Montucla is not altogether fair to the Italian algebraists who preceded Vieta, as to which he has been severely criticised by Cossali, and also by M. Libri. But these Italian historians have a corresponding fault: they make a painful endeavour to show that the peculiar discoveries of Vieta are to be found in the writings of their own illustrious countrymen, and particularly of Cardan. Cossali will even have it that Cardan has something equivalent to, or very nearly approaching to, Des Cartes's theorem on the roots of equations [STURM'S THEOREM]; and constantly endeavours to show that Cardan might, could, would, or should, or ought to have had something which he just stops short of saying Cardan actually *had*. He wants to make his countrymen a school of constructive discoverers; if Cardan had only carried the contents of page *x* farther than he did, and seen something at page *y* which he did not see, then he would have been able at page *z* to do something which he did not do, but which Vieta *did* do. M. Libri starts more fairly: 'In France,' he observes (vol. iv., p. 1), 'Vieta made algebra approach nearer to perfection, and, perhaps, caused the labours of his predecessors to fall into too much neglect.' This is perfectly true, and might have been more positively expressed; but a little further on we find (p. 7), 'In truth his discoveries seem to be not comparable to those of Ferro or Ferrari.' This is truly strange: for in the next sentence we find he 'was an eminently philosophical mind, and is more to be admired for his methods than for the results which he obtained from them.' Can it seriously be M. Libri's opinion that the inventor of an isolated result is to be placed above one who increases the power of the human race over every branch of science? and is it not the surest test of the greatness of a discovery, that it is a method, not a result, and that the power which it gives to others makes succeeding results obtained from it more remarkable than those of the inventor himself. If ever it has been true that coming events have thrown their shadows before, it has been in the progress of the mathematics: it never has happened, in the case of any great discovery, that it was made upon quite a clear field. No one can read the history of science without finding that there was always, in the time immediately preceding the promulgation of any new method, a constant tendency towards the invention of that method, a series of efforts the results of which have speedily merged in those of the man for whom the discovery was reserved. This leaves the relative merit of investigators unaltered; if it depress Vieta, it also depresses Tartaglia and Cardan. To us it raises all three: for it points out that they have severally succeeded where their predecessors have failed, and relieves them from the consequences of the supposition that it was merely their good fortune which led their thoughts to that which another might as easily have attained if his thoughts had been turned towards the subject. If sometimes too much Gallicism shows itself, by way of exception, in the admirable history of Montucla, it is not half so offensive as the constant and always recurring nationality of the Italian historians, which renders it necessary to watch them so closely, that the end of it will be a general conviction that they are not to be safely read at all, without the original authorities at hand, on any matter in which claims of country can enter. M. Libri, in finding out, and with perfect correctness, that Cataldi used continued fractions before Brounker, and infinite series (or at least an infinite series) before Wallis, and in making a very just remark on the interest with which the first dawnings of the doctrine of infinites should be regarded, forgets that Vieta had preceded Cataldi, to the extent of using a combination of the infinite product and series united. It would be difficult, we think, to produce an earlier germ of the doctrine just alluded to than is seen in the celebrated expression given by Vieta for the quadrature of a circle, which we should now express thus

$$\frac{2}{\pi} = \sqrt{a} \, . \, \sqrt{(a + \sqrt{a})} \, . \, \sqrt{\{a + \sqrt{(a + \sqrt{a})}\}} \text{ \&c.}$$

where a means half a unit. (*Resp. Math.*, Schooten, p. 400.)

Both Vieta and Cossali endeavour to show that the Italian algebraists used letters for quantities, both known and unknown. So they did, no doubt, and so did Euclid, and so (according to M. Libri himself) did Aristotle. But who combined the use of letters with that of symbols of operation so as to produce algebraical formulæ, and to give to the operations of algebra that technical character which makes them resemble the operations of arithmetic? One look at any page of the Italian algebraists will show the difference between their algebra and that of Vieta better than any description. Accordingly, both Cossali and Libri state the asserted resemblances without specific citation. When will the writer who asserts that Cardan was substantially in possession of Vieta's algebra attempt to substantiate his assertion by putting so much as half a page of the former side by side with one of the latter?

We now proceed to give some further account of Leonard of Pisa and of Lucas Pacioli, the most celebrated of the very early Italian algebraists. The latter has been accidentally omitted, a circumstance which we do not regret, as it gives us the opportunity of availing ourselves of M. Libri's work hereinbefore cited, and of mentioning the same work in a more satisfactory manner. The author has made most extensive researches in Italian mathematical history, and is, we have no doubt, perfectly trustworthy on all points in which he is not the partisan of a country or a school.

Leonardo Fibonacci (a corruption of *filius Bonaccii*) was the son of one Bonacci, a merchant of Pisa, and was born some time in the twelfth century. He states that his father was employed for the merchants of his own city at the custom-house of an African port, and there made him study arithmetic: he afterwards travelled in Egypt,

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Syria, Greece, and Provence, and from the various systems of numeration which he saw learnt to value the superiority of the Indian method, which was probably that which his father had taught him. His inattention to matters of commerce, and preference for mathematical pursuits, procured for him, from his countrymen, the contemptuous epithet of *Bigollone*. His *Liber Abbaci* was first written in 1202, and with additions in 1228, when it was dedicated to Michael Scott. The *Practica Geometriæ* was written in 1220. Commandine intended to have published the latter, and Bernard the former, but neither effected his purpose, and, with the exception of the parts which Pacioli afterwards used, and the extensive citations in the notes of M. Libri's second volume, nothing of Fibonacci's has yet appeared. There was also a work on square numbers of which the manuscript is known to have existed at Florence in 1768, but cannot now be found.

The *Liber Abbaci* is a work on arithmetic and algebra. M. Libri is of opinion that no Christian writer can be shown to have introduced the Arabic or Indian numerals into any part of Christendom before the publication of this treatise. Such manuscripts as exist, and which seem to have a prior date, are thought by him to have been written either by Jews, or by Spanish Christians among the Moors. Dr. Peacock (*Encycl. Metrop.*, 'Arithmetic') had arrived at the conclusion that Fibonacci's works were the earliest in which these figures can be traced. It is remarkable that their writer was only known by name in the middle of the last century, when the manuscripts of which we now speak were discovered at Florence by Tozzetti. But the intentions of Commandine and Bernard show that they were known at an earlier period.

The fifteenth chapter of the 'Liber Abbaci,' which contains the treatise on algebra, has been cited in full by M. Libri. Any one who will compare it with Dr. Rosen's translation of Mohammed ben Musa will see a resemblance which tends to confirm the general supposition (which also, according to Cardan, may be inferred from the express words of Fibonacci himself) that the Arabic work just named was that from which algebra was made European, though there is every appearance of the avowed translations of it being posterior to Fibonacci. But the latter must either have known other works, or have been an original investigator of great merit. Several things known to the Hindus, but not mentioned by Ben Musa, are contained in his writings. He may have come to these by himself; but it is also certain that the name of the Hindus is frequently mentioned in the manuscripts of the time as that of a nation excelling in these branches of study. A close analysis of the writings of Fibonacci would probably settle whether he is to be considered as having himself enlarged the boundary of the science, or as nothing but the compiler of Oriental works. His merit is great either way; and his name, considerable as it now may be, is nothing to what it will be among the Oriental nations, when they shall have received back the principal which he borrowed from them, with the interest now due upon it, and ready to be paid on demand. The influence of his writings was long felt in Italy, which became from his time the great school of arithmetic; and it is due to him, even now, that his works should be printed entire.

Lucas Pacioli was born at Borgo San Sepolcro, in Tuscany (whence he is frequently called Lucas de Burgo sancti Sepulchri, and Lucas di Borgo), about the middle of the fifteenth century. He was a Minorite friar, and taught successively at Perugia, Rome, Naples, Pisa, and Venice. He resided some time at Milan, in company with Leonardo da Vinci: they quitted Lombardy together on the arrival of the French, and Pacioli spent his last years at Florence and at Venice. He was certainly alive in 1509; but from after that year M. Libri finds no further mention of him as living.

His 'Summa de Arithmetica, Geometria, Proportioni, et Proportionalita' was printed in Italian, at Venice, in 1494. It contains copious extracts from Fibonacci, to such an extent that Pacioli himself warns his reader, where no other authority is mentioned, to infer that Leonard of Pisa is followed. This work was the first printed in algebra, and though it does not advance the science, contains a large amount of details, and carries the practice of algebraical operations into questions of more complexity than any which had preceded, particularly in operations on surd quantities. M. Libri says that the treatise on book-keeping, which forms part of Pacioli's work, is the fi[illegible] which what is now called the method of double entr[illegible] pears in print. Some account of the contents w[illegible] found in Hutton's 'History of Algebra' (*Tracts*, vol [illegible] The 'Divina Proportione,' Venice, 1509, is thus descr[illegible] by M. Libri: 'Pacioli wished to make a certain pro[illegible]tion,* long known to geometers, the base of a[illegible] sciences. He deduces from it the principles of arch[illegible]ture, the proportions of the human figure, and even t[illegible] which ought to be given to the letters of the alphabet[illegible] is a systematic treatise, of which the principal merit c[illegible]sists in the co-operation of Leonardo da Vinci, who [illegible]graved the plates, and probably also superintended [illegible] parts which concern the arts. There are some prop[illegible]tions of geometry upon the inscription of polyhedra in [illegible] another....There is also the use of letters to ind[illegible] numerical quantities.' On this last sentence M. Libri [illegible] a passage containing the use of letters in a simple pro[illegible]tion; and it seems to us that the point which he is labo[illegible]ing to establish, namely, the virtual existence of *spec*[illegible] algebra before Vieta, cannot be more completely ove[illegible]turned than by this, his only direct quotation on the su[illegible]ject. When M. Libri says that Fibonacci used letters [illegible] quantities, both known and unknown, he does not cite a passage, but leaves it to be verified by those who will loo[illegible] over his citation of the fifteenth chapter of Fibonacci[illegible] more than 150 octavo pages. On looking through th[illegible] we do find a few places where numbers are denoted [illegible] single letters; but whenever they are to be divided [illegible] parts, double letters are used: in fact, Fibonacci [illegible] exactly what Euclid does in the fifth book. Of Pac[illegible] notation, in the professed algebraical work, nothing [illegible] said; but in the work we now mention the quota[illegible] which is to establish that Pacioli had substantially [illegible] idea of Vieta on algebra contains just as much algebra[illegible] notation as, and no more than, appears in Pacioli's [illegible] translation of Euclid, published in the same year. M[illegible] Libri persists in supposing that the mere use of le[illegible] to designate numbers is the sole distinction of Vi[illegible] algebra.

The edition of Euclid, to which we have just allu[illegible] and which appeared in Latin, at Venice, in 1509, is t[illegible] to which [GEOMETRY, p. 155] we have followed the [illegible] of Fabricius in doubting its existence. We have since [illegible] the work. Heilbronner infers from the preface to [illegible] 'Divina Proportione,' that Pacioli translated Euclid [illegible] Italian, and it is now known that he did not publish sev[illegible] of his earlier works: but he himself, in the dedicatio[illegible] the work now under mention, speaking of this very [illegible] Euclid itself, says, 'Leges....vernacula lingua per [illegible] donatum Euclidem:' whence it is obvious that by *ver*[illegible]*cula* he means the Latin, as opposed to Greek or Ara[illegible] The translation is substantially that of Athelard ([illegible] goes by the name of Campanus), and the commentar[illegible] Campanus, or many of them, are added: Pacioli's [illegible] additional comments are all headed *Castigator*. All [illegible] fifteen books are given which were supposed to [illegible] Euclid's.

Pacioli is not to be looked on as a great improver [illegible] of geometry or arithmetic: but his utility cannot [illegible] denied. It was he who made Fibonacci useful to [illegible] world by his compilations from that writer, and he [illegible] shown so much learning on the subject, and has d[illegible] from so many sources, that it is not perhaps too much [illegible] say that it was better he should have printed the [illegible] book on algebra, than a more original but less er[illegible] teacher.

VIGA GANITA, the name of the principal Hi[illegible] work on algebra which remains. We have referred to [illegible] article all matters which relate to the astronomical [illegible] arithmetical science of the Hindus, partly because th[illegible] is not enough to be said on the subject to make it w[illegible] while to distribute it under heads in a work like the [illegible]sent, and partly because it was desirable to defer the [illegible] in question as long as possible, in the hope that [illegible] further investigation of the points on which we ar[illegible] write might make its appearance. For it is not a s[illegible] record of facts, but an account of the most singular [illegible]tremes of opinion, which is to be given, almost every p[illegible] having been discussed in the most extreme spirit of p[illegible]

* We cite it here because we wish to give some account of a work w[illegible] usually only mentioned, and because we cannot understand what M. L[illegible] means.

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